Note on "Risk, Unemployment, and the Stock Market: A Rare-Event-Based Explanation of Labor Market Volatility"

by Kilic and Wachter (2018)

Abstract

This note presents a calibration for the model in Kilic and Wachter (2018) that employs a standard matching function ensuring vacancy-filling probabilities lie between zero and one, together with an explicit non-negativity constraint on vacancy openings. This specification corrects an error in the original calibration related to states with negative vacancy openings, which generates inconsistent implications for the equity premium and the government bill rate. The proposed calibration delivers the desired quantitative implications while preserving the model's underlying economic mechanism.

1 Calibration and Results

In this section, we describe the calibration and quantitative results. The notation and other model assumptions are identical to those in Kilic and Wachter (2018). The Appendix includes the model derivation using the non-negativity constraint on V_t and details of numerical computation.

1.1 Parameters

Table 1 reports parameter values. The following parameters are identical to the values reported in Kilic and Wachter (2018): disaster size distribution ζ, productivity growth μ, productivity volatility σ_ϵ, bargaining power B, value of non-market activity b, government default probability q. We set the time preference parameter β to 0.998, risk aversion γ to 1.95, and the elasticity of intertemporal substitution (EIS) to 1.15 jointly considering model implications for the equity premium and government bill rate. The calibration originally presented in Kilic and Wachter (2018) assumes a higher risk aversion (4.9) and EIS (2) which, using an accurate solution, imply a too low government bill rate and a too high equity premium. As illustrated in this note, the modified model performs well with lower values for these parameters.

The separation rate s is set to 0.025 which is slightly below values used in the literature and lies between the empirical gross separation rates and quit rates from Davis, Faberman, and Haltiwanger (2006). Our choice of the tightness insulation parameter ν is in line with the empirical evidence on the elasticity of wages to labor market tightness reported in Table 2. We use the matching function $m(U_t, V_t) = \frac{U_t V_t}{(U_t^\iota + V_t^\iota)^{1/\iota}}$. The matching function parameter ι is set to 0.75 which lies between the values used by Den Haan, Ramey, and Watson (2000) and Hagedorn and Manovskii (2008). We approximate the dynamics of λ_t using a 14-state Markov chain . The persistence of λ_t is calibrated to the persistence of unemployment in the data, and the volatility is bounded by the constraint $\lambda_t \leq 1$. Table 3 presents the states for λ_t and Table 4 shows that the simulated dynamics of λ_t are similar to those used in Kilic and Wachter (2018). Finally, the dynamics of $\hat{\kappa}$ target the behavior of unemployment during the Great

Depression given the policy functions implied by the other parameters as shown in Figure 2.

1.2 Simulation Results

This section presents quantitative results from the model. We show that the economic mechanism as well as the quantitative results are close to those reported in Kilic and Wachter (2018). We simulate the model at monthly frequency and report results from a long simulation (Population) with length 100,000 years as well as 10,000 simulations with length 60 years. "No-Disaster" refers to simulations in which there is no disaster realization and the quarterly averages of monthly vacancies are always positive.[1] 48% of all simulations do not have a disaster realization and in 41% of all simulations, quarterly vacancies are positive and there is no disaster realization.

Figure 3 shows the response of model quantities to an increase of monthly disaster probability λ_t. Consumption initially rises and then falls to a level below the level before the increase in λ_t as employment drops due to a level level of vacancy openings. The price-productivity ratio also decreases upon the increase in λ_t consistent with the mechanism proposed by Kilic and Wachter (2018). Figure 4 shows that the equity return reacts negatively to the increase in λ_t and the equity premium increases upon the increase in λ_t.

Table 5 shows that labor market tightness predicts excess equity market returns with a negative sign as in the data. Table 6 reports labor market statistics and shows that no-disaster samples from the model explain most of the volatility in vacancies, unemployment and labor market tightness. Table 7 reports business cycle and financial moments. The model implies a realistically low volatility for consumption and output growth as well as the government bill rate. The equity return volatility is close to the data at the median of the no-disaster samples and the median equity premium is higher than the data while the empirical value is within the confidence bands of the model. Finally, Figure 5 shows that the model produces a wide range of values for unemployment and vacancies consistent with the Beveridge curve in the data. The average unemployment rate in simulations is 8.5% which is higher

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[1] We follow the literature and report volatilities of log deviations from an HP trend. Since the constraint $V_t \geq 0$ is binding in 2.45% of months in population and the logarithm of zero is minus infinity, we do not report population results in Table 6.

than the average in the postwar average civilian unemployment rate and closer to the private nonfarm unemployment rate reported in Petrosky-Nadeau and Zhang (2019).

Table 8 and Table 9 report labor market volatility and financial moments in special cases of the model with no tightness insulation of wages, constant vacancy costs and constant disaster probability. The results illustrate the contribution of each model assumption to the quantitative results and confirm that the baseline model is the most consistent with the data.

Finally, Tables 10 and 11 report results from the extension of the model with capital and show that our results on labor market volatility and financial moments are robust to inclusion of physical capital in the model. In this extension, we keep the parameters related to capital ($\delta = 0.01$, $r = -0.5385$, $\alpha = 0.65$, $\xi_k = -0.5385$) identical to those in Kilic and Wachter (2018).

References

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Table 1: Parameters Values for Monthly Calibration

Parameter	Value
Time preference, β	0.998
Risk aversion, γ	1.95
Elasticity of intertemporal substitution, ψ	1.15
Disaster size distribution (GDP), ζ	multinomial
Productivity growth, μ	0.0018
Productivity volatility, σ_ϵ	0.0047
Matching function parameter, ι	0.75
Separation rate, s	0.025
Bargaining power, B	0.5
Value of nonmarket activity, b	0.4
Vacancy cost in normal times, $\underline{\kappa}$	0.78
Vacancy cost in disasters, $\overline{\kappa}$	0.94
Vacancy cost persistence, ρ_κ	0.95
Tightness insulation, ν	0.03
Government default probability, q	0.4

Table 2: Properties of Aggregate Wages

	SD	AC	$\epsilon_{W,\theta}$	$\epsilon_{W,Z}$	$\epsilon_{\theta,Z}$
Panel A: Data					
1951 - 2013	1.77	0.91	0.00	0.67	2.46
	—	—	[0.33]	[5.43]	[0.76]
1951 - 1985	1.21	0.91	0.01	0.35	11.22
	—	—	[2.75]	[3.04]	[3.86]
1986 - 2013	2.29	0.91	-0.01	1.07	-8.49
	—	—	[-1.15]	[6.79]	[-2.37]
Panel B: Benchmark model					
50%	1.65	0.92	0.00	1.00	-0.05
5%	1.30	0.87	-0.02	0.98	-5.21
95%	2.10	0.95	0.03	1.02	5.12
Panel C: No tightness insulation of wages					
50%	1.72	0.91	0.27	1.00	0.00
5%	1.36	0.87	-0.11	0.92	-0.31
95%	2.17	0.95	0.63	1.08	0.31

Notes: SD denotes standard deviation, AC quarterly autocorrelation. Z is labor productivity, θ labor market tightness. Data are from 1951 to 2013. All data and model moments are in quarterly terms. We simulate 10,000 samples with length 60 years at monthly frequency and report quantiles from 41% of simulations that include no disaster realization and no quarterly vacancies equal to zero. $\epsilon_{x,y}$ is the elasticity of variable x to y, namely, the regression coefficient of log x on log y. Data t-statistics in brackets are based on Newey-West standard errors. All variables are used in logs as deviations from an HP trend with smoothing parameter 10^5.

Table 3: Monthly Disaster Probability

Value	Stationary Probability
1 x 10^{-6}	0.0001
5 x 10^{-6}	0.0016
2 x 10^{-5}	0.0095
0.0001	0.0349
0.0003	0.0873
0.0012	0.1571
0.0050	0.2095
0.0200	0.2095
0.0801	0.1571
0.3204	0.0873
1.2820	0.0349
5.1305	0.0095
20.5314	0.0016
82.1632	0.0001

Notes: Table lists the nodes of a 14-state Markov process which approximates an AR(1) process for log probabilities. Disaster probabilities are in percentage terms.

Table 4: Monthly Disaster Probability in Simulations

		No-Disaster				All Simulations			
	Population	Mean	5%	50%	95%	Mean	5%	50%	95%
$\mathbb{E}[\lambda]$	0.19	0.06	0.01	0.05	0.13	0.18	0.02	0.10	0.63
$\sigma(\lambda)$	1.49	0.14	0.02	0.12	0.33	0.61	0.04	0.25	2.47
$\rho(\lambda)$	0.94	0.89	0.75	0.90	0.96	0.90	0.77	0.91	0.97

Notes: σ denotes volatility, ρ monthly autocorrelation. Disaster probabilities are in percentage terms. Population is a sample of 100,000 years. We simulate 10,000 samples with length 60 years at monthly frequency and report statistics from all simulations as well as from 41% of simulations that include no disaster realization and no quarterly vacancies equal to zero. All simulations are in monthly frequency.

Table 5: Excess return predictability by labor market tightness

Months	1	6	12	24	36	60
Panel A: Data						
β_θ	-0.13	-0.12	-0.10	-0.07	-0.07	-0.06
t-statistic	[-3.33]	[-3.82]	[-3.51]	[-3.10]	[-3.33]	[-4.33]
R^2	0.02	0.07	0.10	0.11	0.14	0.23
Panel B: β_θ in the model						
Median, no-disaster samples	-0.04	-0.03	-0.03	-0.02	-0.02	-0.01
Population	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
Panel C: R^2 in the model						
Median, no-disaster samples	0.18	0.63	0.79	0.77	0.65	0.45
Population	0.06	0.23	0.30	0.31	0.27	0.20

Note: Table reports results from predictability regressions of the form

$$\frac{1}{n}\left(\log(R_{t,t+n}) - \log(R^b_{t,t+n})\right) = \beta_0 + \beta_\theta \log(\theta_t) + \epsilon_{t,t+n},$$

where n is the predictability horizon in months and data are at the monthly frequency. Data t-statistics are based on Newey-West standard errors. We simulate 10,000 samples with length 60 years from the model and report quantiles from 41% of simulations that include no disaster realization and no quarterly vacancies equal to zero. Population values are from a path with length 100,000 years.

Table 6: Labor Market Moments

	U	V	V/U	Z	P/Z	
Panel A: Data						
SD	0.19	0.21	0.39	0.02	0.16	
AC	0.94	0.94	0.95	0.88	0.89	
	1	-0.86	-0.96	-0.18	-0.44	U
	—	1	0.97	0.03	0.47	V
	—	—	1	0.10	0.47	V/U
	—	—	—	1	0.00	Z
	—	—	—	—	1	P/Z
Panel B: No-Disaster Simulations						
SD	0.13	0.17	0.27	0.02	0.09	
	(0.06)	(0.09)	(0.13)	(0.00)	(0.04)	
AC	0.94	0.66	0.84	0.91	0.89	
	(0.02)	(0.10)	(0.06)	(0.02)	(0.04)	
	1.00	-0.56	-0.85	0.01	-0.91	U
	—	1.00	0.91	-0.00	0.84	V
	—	—	1.00	-0.01	0.98	V/U
	—	—	—	1.00	-0.01	Z
	—	—	—	—	1.00	P/Z

Notes: SD denotes standard deviation, AC quarterly autocorrelation. Data are from 1951 to 2013. All data and model moments are in quarterly terms. U is unemployment, V vacancies, Z labor productivity and P/Z price-productivity ratio. We simulate 10,000 samples with length 60 years at monthly frequency and report means from 41% of simulations that include no disaster realization and no quarterly vacancies equal to zero in Panel B. Standard errors across simulations are reported in parentheses. Standard deviations, autocorrelations and the correlation matrix are calculated using log deviations from an HP trend with smoothing parameter 10^5.

Table 7: Business Cycle and Financial Moments

	$\mathbb{E}[\Delta c]$	$\mathbb{E}[\Delta y]$	$\sigma(\Delta c)$	$\sigma(\Delta y)$	$\mathbb{E}[R - R_b]$	$\mathbb{E}[R_b]$	$\sigma(R)$	$\sigma(R_b)$
Data	1.97	1.90	1.78	2.29	5.32	1.01	12.26	2.22
Simulation %50	2.17	2.17	1.84	1.90	9.10	1.44	11.46	3.27
Simulation %5	1.80	1.79	1.29	1.32	4.86	-0.38	5.52	1.45
Simulation %95	2.55	2.55	4.77	4.81	15.41	2.73	19.88	6.47
Population	1.65	1.65	10.72	10.75	13.08	0.02	26.06	8.52

Notes: The table reports means and volatilities of log consumption growth (Δc), log output growth (Δy), the government bill rate (R_b) and the unlevered equity return R in historical data and in data simulated from the model. All data and model moments are in annual terms. Historical data are from 1951-2013. We simulate 10,000 samples with length 60 years from the model and report quantiles from 41% of simulations that include no disaster realization and no quarterly vacancies equal to zero. Population values are from a path with length 100,000 years.

Table 8: Comparative Statics for Labor Market Volatility

	U	V	V/U	P/Z
Data	0.19	0.21	0.39	0.16
Benchmark	0.13	0.17	0.27	0.09
No tightness insulation of wages	0.01	0.01	0.02	0.01
Constant scaled-vacancy cost $\hat{\kappa}$	0.05	0.05	0.09	0.04
Constant disaster probability λ	0.00	0.00	0.01	0.00
Constant $\hat{\kappa}$ and λ	0.00	0.00	0.00	0.00

Notes: Standard deviations (in log deviations from an HP trend) for unemployment (U), vacancies (V), labor productivity (Z) and the price-productivity ratio (P/Z) in the data and in four versions of the model. Data are from 1951 to 2013. All data and model moments are in quarterly terms. Model values are calculated by simulating 10,000 samples with length 60 years at a monthly frequency. We report means from 41% of simulations that include no disaster realization and no quarterly vacancies equal to zero. In the constant disaster probability model, we set disaster probability to 0.18%. In the constant $\hat{\kappa}$ model, vacancy costs are assumed to be constant at its lower bound.

Table 9: Comparative Statics for Business Cycle and Financial Moments

	$\mathbb{E}[\Delta c]$	$\mathbb{E}[\Delta y]$	$\sigma(\Delta c)$	$\sigma(\Delta y)$	$\mathbb{E}[R - R_b]$	$\mathbb{E}[R_b]$	$\sigma(R)$	$\sigma(R_b)$
Data	1.97	1.90	1.78	2.29	5.32	1.01	12.26	2.22
Panel A: Benchmark								
No-disaster median	2.17	2.17	1.84	1.90	9.10	1.44	11.46	3.27
Population	2.55	1.65	10.72	10.75	13.08	0.02	26.06	8.52
Panel B: No tightness insulation of wages								
No-disaster median	2.16	2.16	1.32	1.32	-0.28	0.82	4.11	3.45
Population	1.64	1.64	4.98	4.97	-0.26	-0.60	7.08	7.03
Panel C: Constant scaled-vacancy cost $\hat{\kappa}$								
No-disaster median	2.16	2.16	1.38	1.42	-2.03	0.58	4.56	3.27
Population	1.64	1.64	5.13	5.13	-1.49	-0.75	6.12	6.74
Panel D: Constant disaster probability λ								
No-disaster median	2.17	2.17	1.33	1.33	1.58	3.92	1.72	0.00
Population	1.65	1.65	7.41	7.59	0.77	3.71	8.31	3.68
Panel E: Constant $\hat{\kappa}$ and λ								
No-disaster median	2.16	2.16	1.31	1.31	0.68	3.80	1.69	0.00
Population	1.67	1.67	3.79	3.79	0.43	3.62	3.90	2.27
Panel F: No disaster risk								
No-disaster median	2.16	2.16	1.32	1.32	0.06	4.34	1.69	0.00
Population	2.15	2.15	1.33	1.33	0.05	4.34	1.70	0.00

Notes: Δc denotes log consumption growth, Δy log output growth, R the unlevered equity return, R_b the government bill rate. All data and model moments are in annual terms. We simulate 10,000 samples with length 60 years at monthly frequency and report the median from 41% of simulations that include no disaster realization and no quarterly vacancies equal to zero. In the constant disaster probability model, we set disaster probability to 0.18%, the stationary mean of the disaster probability process used in the benchmark model. In the constant κ model, vacancy costs are assumed to be constant at its lower bound. Population values are from a path with length 100,000 years. Returns and growth rates are aggregated to annual values.

Table 10: Labor Market Moments in the Model with Capital

	U	V	V/U	Z	P/Z	
Panel A: Data						
SD	0.19	0.21	0.39	0.02	0.16	
AC	0.94	0.94	0.95	0.88	0.89	
	1	-0.86	-0.96	-0.18	-0.44	U
	—	1	0.97	0.03	0.47	V
	—	—	1	0.10	0.47	V/U
	—	—	—	1	0.00	Z
	—	—	—	—	1	P/Z
Panel B: No-Disaster Simulations						
SD	0.13	0.16	0.26	0.02	0.10	
	(0.06)	(0.08)	(0.12)	(0.00)	(0.03)	
AC	0.94	0.67	0.85	0.91	0.89	
	(0.02)	(0.10)	(0.05)	(0.02)	(0.03)	
	1.00	-0.57	-0.86	0.00	-0.86	U
	—	1.00	0.91	-0.00	0.83	V
	—	—	1.00	-0.00	0.95	V/U
	—	—	—	1.00	-0.00	Z
	—	—	—	—	1.00	P/Z

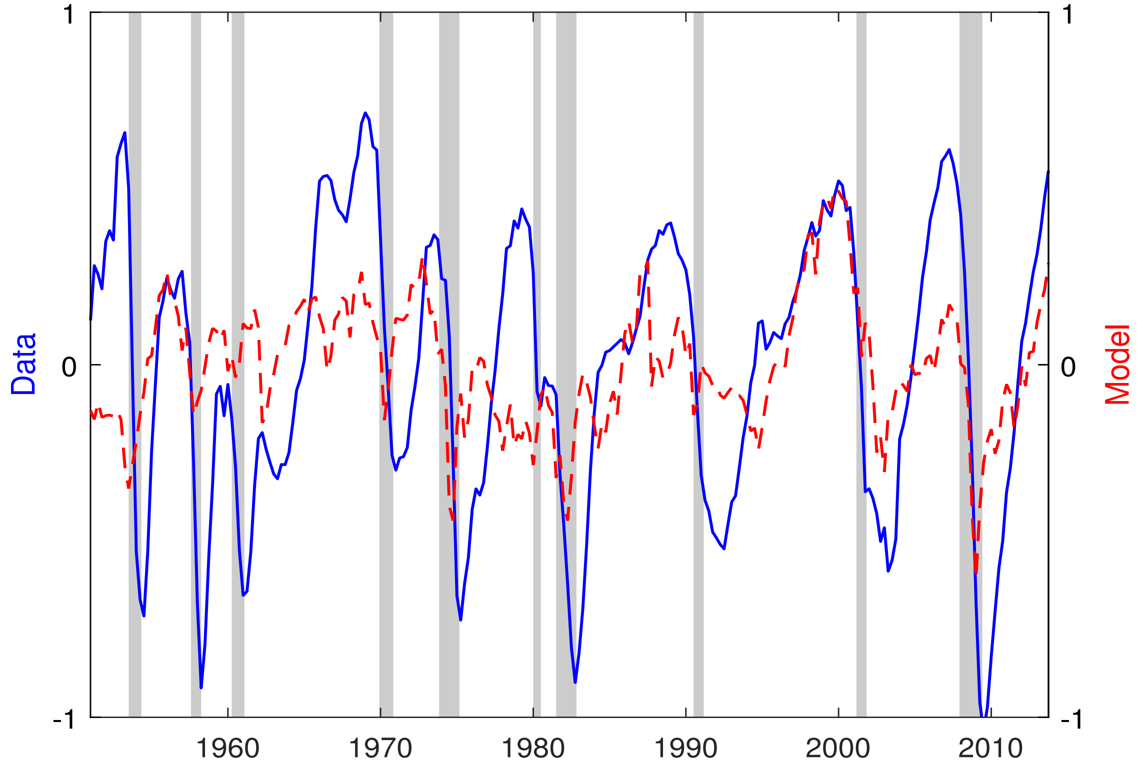
Notes: SD denotes standard deviation, AC quarterly autocorrelation. All model moments are in quarterly terms. U is unemployment, V vacancies, Z labor productivity and P/Z price-productivity ratio. We simulate 10,000 samples with length 60 years at monthly frequency and report means from 41% of simulations that include no disaster realization and no quarterly vacancies equal to zero. Standard errors across simulations are reported in parentheses. Population values in Panel C are from a path with length 100,000 years at monthly frequency. Standard deviations, autocorrelations and the correlation matrix are calculated using log deviations from an HP trend with smoothing parameter 10^5.

Table 11: Business Cycle and Financial Moments in the Model with Capital

	$\sigma(\Delta c)$	$\sigma(\Delta i)$	$\sigma(\Delta y)$	$\mathbb{E}[R - R_b]$	$\mathbb{E}[R_b]$	$\sigma(R)$	$\sigma(R_b)$
Data	1.78	8.76	2.29	5.32	1.01	12.26	2.22
Simulation %50	1.62	4.70	1.86	10.50	1.79	13.14	3.17
Simulation %5	1.24	3.14	1.31	7.16	-0.08	8.10	1.38
Simulation %95	3.40	7.07	4.59	15.30	3.04	21.33	6.23
Population	7.77	9.68	10.32	13.80	0.37	32.37	8.05

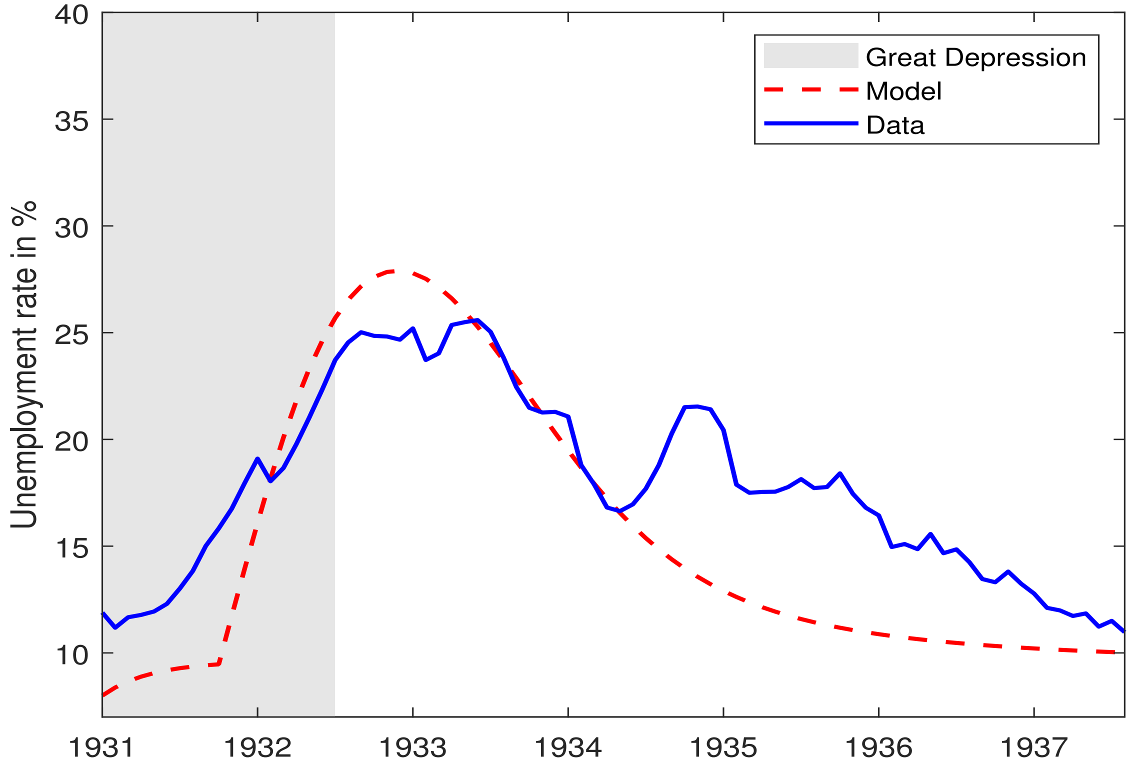
Notes: The table reports means and volatilities of log consumption growth (Δc), log investment growth (Δi), log output growth (Δy), the government bill rate (R_b) and the unlevered equity return R in historical data and in data simulated from the model. All data and model moments are in annual terms. Historical data are from 1951-2013. We simulate 10,000 samples with length 60 years from the model and report quantiles from 41% of simulations that include no disaster realization and no quarterly vacancies equal to zero. Population values are from a path with length 100,000 years.

Figure 1: Vacancy-Unemployment Ratio: Data vs. Model



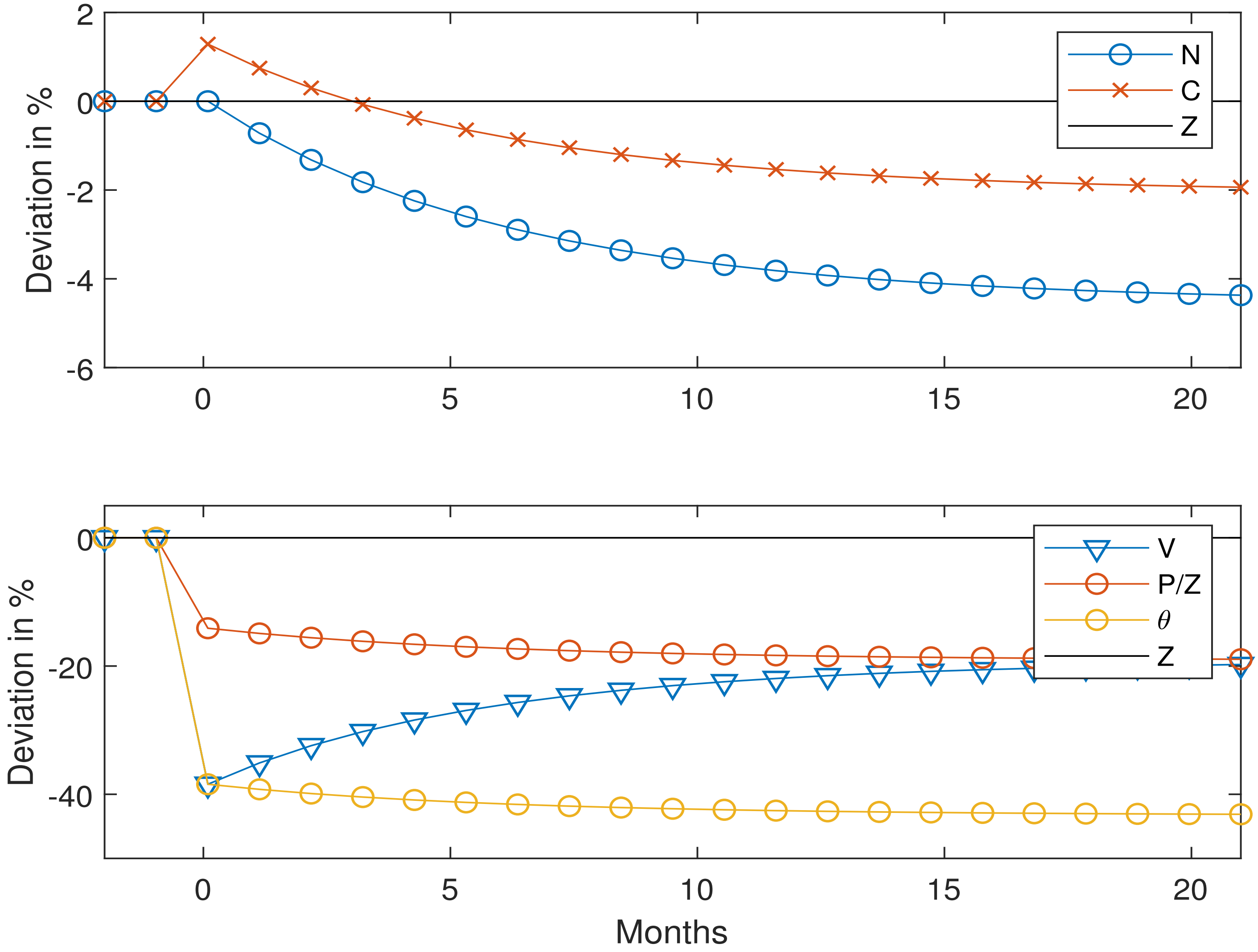
Notes: The solid line and the dashed line show the vacancy-unemployment ratio in the data and in the model, respectively. Model-implied vacancies are calculated by substituting the price-productivity ratio and employment level from the data into equation (A.7), assuming labor-market parameters given in Table 1. Values are log deviations from an HP trend with smoothing parameter 10^5. Shaded periods are NBER recessions.

Figure 2: Unemployment in the Great Depression



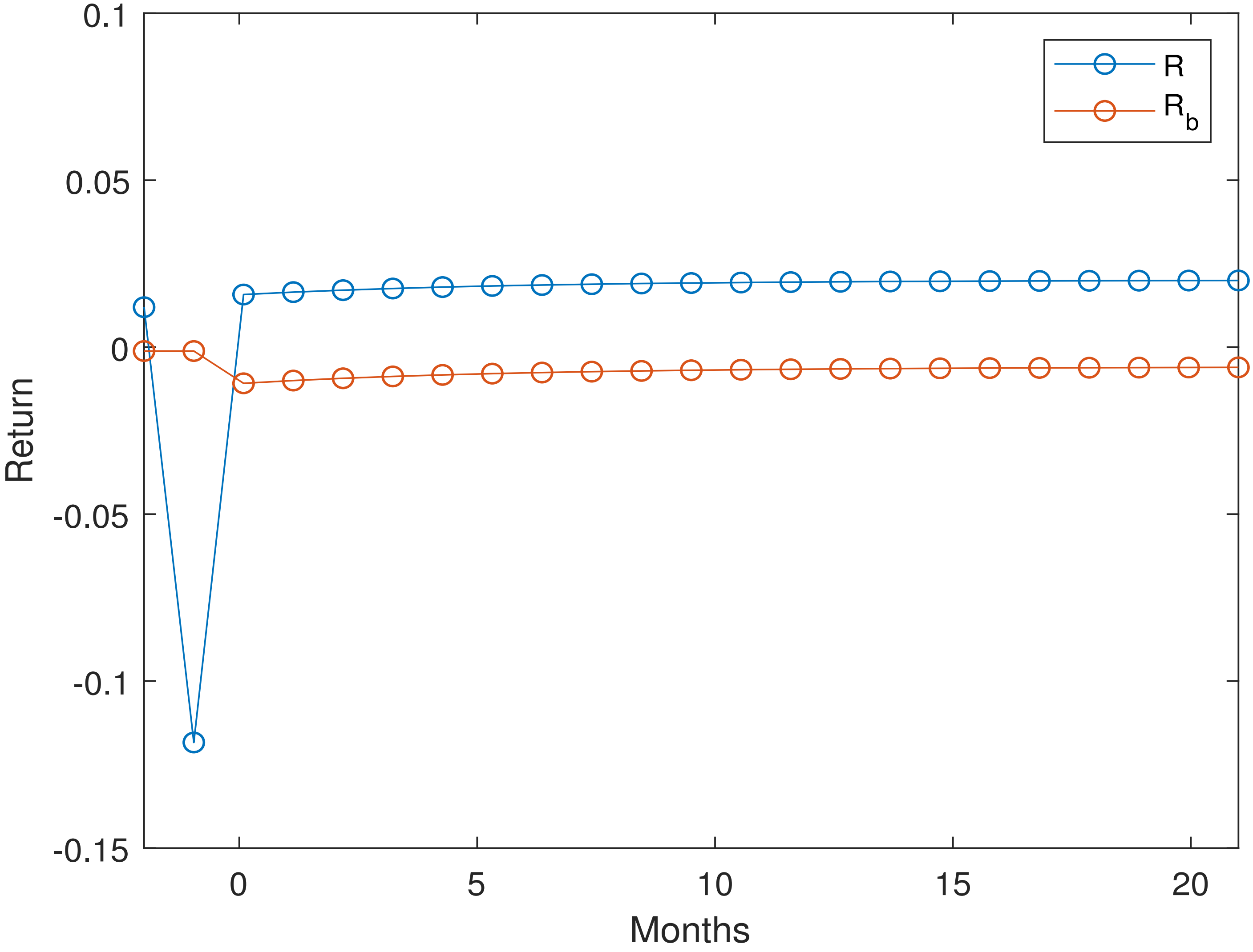
Notes: Figure plots the unemployment rate from 1931 to 1937 in the data and implied by the model. Disaster probability is assumed to be 0.08% throughout. The disaster realization is assumed to be at the end of 1931.

Figure 3: Macroeconomic Response to Increase in Disaster Probability



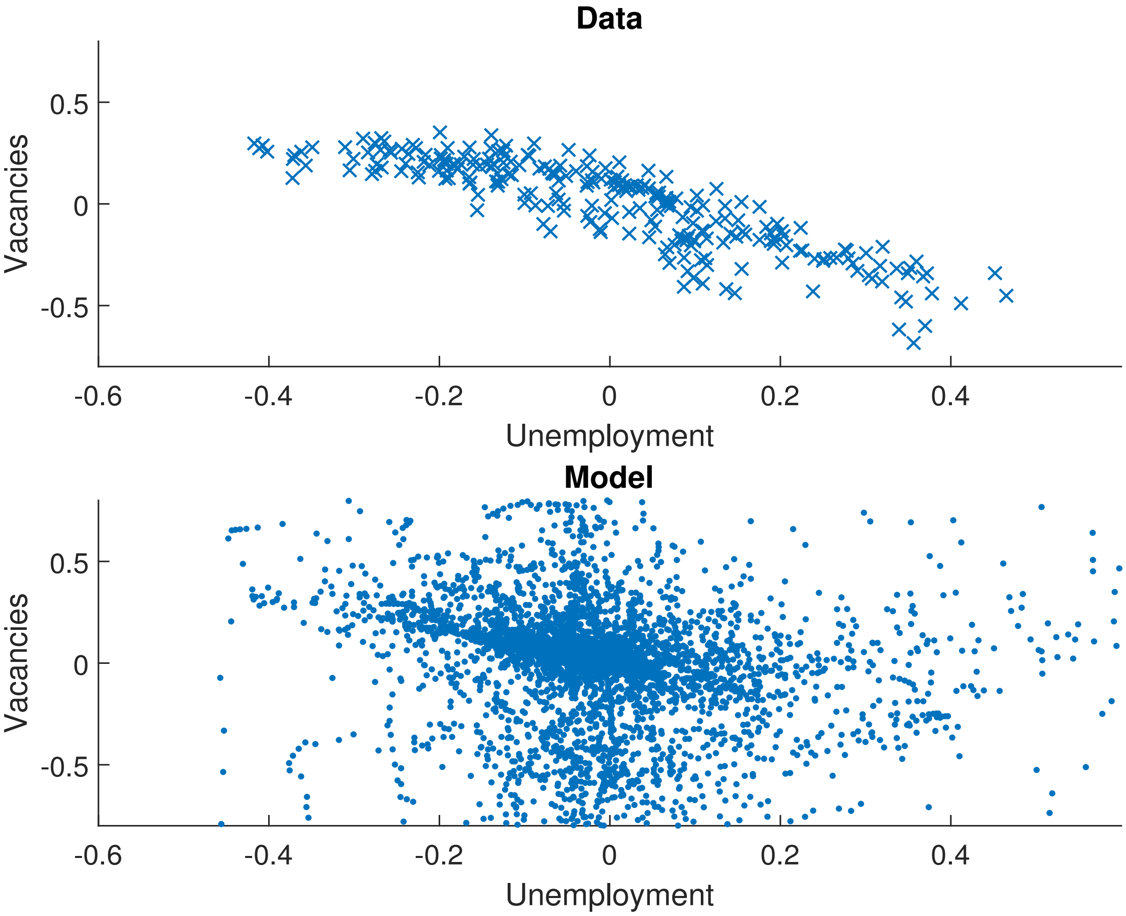
Notes: In month zero, monthly disaster probability increases from 0.08% to 0.32% and stays at 0.32% in the remaining months.

Figure 4: Return Response to Increase in Disaster Probability



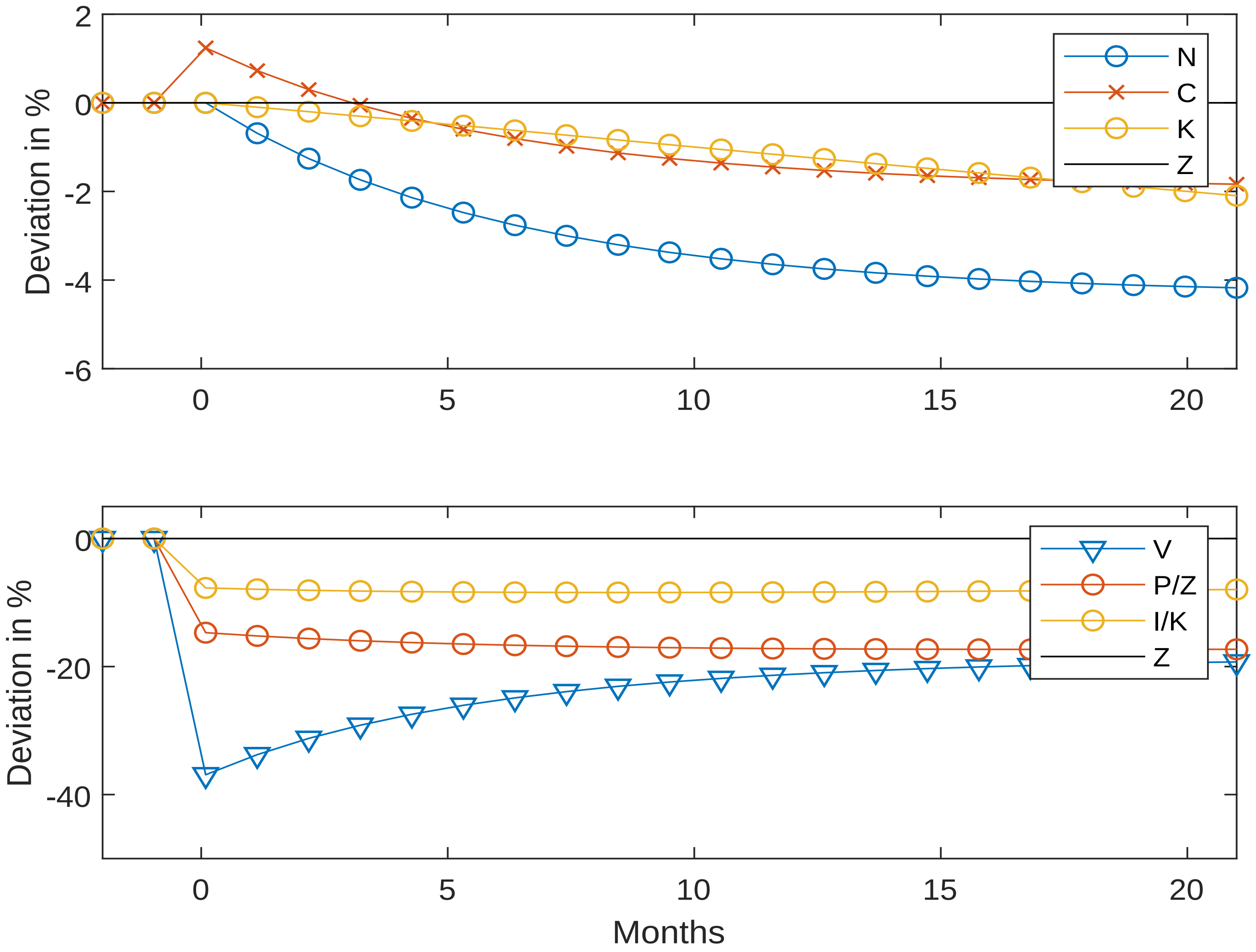
Notes: In month zero, monthly disaster probability increases from 0.08% to 0.32% and stays at 0.32% in the remaining months.

Figure 5: Beveridge Curve



Notes: Data are quarterly from 1951 to 2013. Model implied curve is a quarterly sample with length 10,000 quarters from the stationary distribution. All values are log deviations from an HP trend with smoothing parameter 10^5.

Figure 6: Macroeconomic Response to Increase in Disaster Probability in the Model with Capital



Notes: In month zero, monthly disaster probability increases from 0.08% to 0.32% and stays at 0.32% in the remaining months.

Appendix

A Model solution

In this section, we describe two modifications to the model in Kilic and Wachter (2018) that facilitate a stable and accurate numerical solution: a different functional form for the matching function and a non-negativity constraint on vacancies V_t. The notation and other model assumptions are identical to those in Kilic and Wachter (2018).

A.1 Matching Function

We adopt the following labor matching function used in Den Haan, Ramey, and Watson (2000), Hagedorn and Manovskii (2008) and Petrosky-Nadeau, Zhang, and Kuehn (2018):

$$m(U_t, V_t) = \frac{U_t V_t}{(U_t^{\iota} + V_t^{\iota})^{1/\iota}}, \tag{A.1}$$

where $\iota > 0$. This functional form features the constant returns to scale property of the Cobb-Douglas matching function. Define labor market tightness $\theta_t = V_t/U_t$. The vacancy filling rate is $q(\theta_t) = m(U_t, V_t)/V_t = (1 + \theta_t^{\iota})^{-1/\iota}$. Unlike the Cobb-Douglas matching function, equation (A.1) guarantees that the vacancy filling probability $q(\theta_t)$ is between zero and one. Furthermore, $q(\theta_t)$ is decreasing in θ_t and $q(0) = 1$.

A.2 Non-negativity constraint $V_t \geq 0$

The search and matching model does not entertain the possibility of negative vacancy openings and the maximum drop in employment per period is driven by the separation rate. Theoretically, it is possible to obtain model solutions where vacancies remain positive in all states on a grid of state variables. This does not guarantee, however, that vacancies are positive in all predicted future states leading to potential inaccuracies in the numerical solution.

We solve the model with the constraint $V_t \geq 0$ which is equivalent to $Z_t q(\theta_t) V_t \geq 0$ because $q(\theta_t) > 0$

and $Z_t > 0$. In what follows, we derive Theorem 1 in Kilic and Wachter (2018) incorporating the non-negativity constraint and show that the solution is identical to that without the non-negativity constraint up to an adjustment of the shadow value of labor inside the firm.

The firm maximizes the present value of current and future dividends

$$\max_{\{V_{t+\tau}, N_{t+\tau+1}\}_{\tau=0}^{\infty}} \mathbb{E}_t \sum_{\tau=0}^{\infty} M_{t+\tau} D_{t+\tau} \tag{A.2}$$

subject to

$$N_{t+1} = (1-s)N_t + q(\theta_t)V_t, \tag{A.3}$$

where $q(\theta_t) = (1+\theta_t^{\iota})^{-1/\iota}$, and

$$Z_t q(\theta_t) V_t \geq 0. \tag{A.4}$$

Theorem A.2.1. *Assume the production function $Y_t = Z_t N_t$ and that the firm solves (A.2). Then the ex-dividend value of the firm is given by*

$$P_t = \left(\frac{\kappa_t}{q(\theta_t)} - l_t^v Z_t \right) N_{t+1}, \tag{A.5}$$

where l_t^v is the Lagrange multiplier on the constraint (A.4) and the equity return equals

$$R_{t+1} = \frac{(1-s)\left(\frac{\kappa_{t+1}}{q(\theta_{t+1})} - l_{t+1}^v Z_{t+1}\right) + Z_{t+1} - W_{t+1}}{\frac{\kappa_t}{q(\theta_t)} - l_t^v Z_t}. \tag{A.6}$$

Furthermore, if $\kappa_t = \kappa Z_t$ for fixed κ, then

$$\frac{P_t}{Z_t} = \left(\frac{\kappa}{q(\theta_t)} - l_t^v \right) N_{t+1}. \tag{A.7}$$

Proof of Theorem A.2.1 The representative firm pays out as dividend what is left from output after subtracting wage costs and investment in hiring:

$$D_t = Z_t N_t - W_t N_t - \kappa_t V_t. \tag{A.8}$$

The firm takes wages W_t and labor market tightness θ_t as given and maximizes the cum-dividend value

$$P_t^c = \max_{\{V_{t+\tau}, N_{t+\tau+1}\}_{\tau=0}^{\infty}} \mathbb{E}_t \sum_{\tau=0}^{\infty} M_{t+\tau} \left[Z_{t+\tau} N_{t+\tau} - W_{t+\tau} N_{t+\tau} - \kappa_{t+\tau} V_{t+\tau} \right], \tag{A.9}$$

subject to the law of motion for employment

$$N_{t+1} = (1-s)N_t + q(\theta_t)V_t. \tag{A.10}$$

and the non-negativity constraint for vacancies

$$Z_t q(\theta_t)V_t \geq 0. \tag{A.11}$$

The Kuhn-Tucker condition that optimal policies need to satisfy is given by

$$l_t^v Z_t q(\theta_t) V_t = 0. \tag{A.12}$$

The first-order conditions with respect to V_t and N_{t+1} are given by

$$\kappa_t = l_t^v Z_t q(\theta_t) + l_t^n q(\theta_t), \tag{A.13}$$

$$l_t^n = \mathbb{E}_t \left[M_{t+1}(Z_{t+1} - W_{t+1} + l_{t+1}(1-s)) \right], \tag{A.14}$$

where l_t^n is the Lagrange multiplier on the aggregate law of motion for employment level (A.10) and l_t^v is the Lagrange multiplier on the non-negativity constraint for vacancies (A.11). Note that (A.14) can be interpreted as an Euler equation with l_t^n as the value of a worker inside the firm.

We expand (A.9), adding to each term in the summation an expression that, by (A.10) and (A.12), is equal to zero:

$$\begin{aligned} P_t^c &= Z_t N_t - W_t N_t - \kappa_t V_t - l_t^n \left(N_{t+1} - (1-s)N_t - q(\theta_t)V_t\right) + l_t^v Z_t q(\theta_t) V_t \\ &+ \mathbb{E}_t \left[M_{t+1} \left[Z_{t+1}N_{t+1} - W_{t+1}N_{t+1} - \kappa_{t+1}V_{t+1} - l_{t+1}^n \left(N_{t+2} - (1-s)N_{t+1} - q(\theta_{t+1})V_{t+1}\right) + l_{t+1}^v Z_{t+1} q(\theta_{t+1}) V_{t+1} \right]\right] \\ &+ \ldots \end{aligned} \tag{A.15}$$

The terms $-\kappa_t V_t$ and $l_t^n q(\theta_t)V_t + l_t^v Z_t q(\theta_t) V_t$ cancel out for all t as a result of (A.13). Furthermore, $l_t^n N_{t+1}$ cancels out with $\mathbb{E}_t \left[Z_{t+1}N_{t+1} - W_{t+1}N_{t+1} + l_{t+1}(1-s)N_{t+1}\right]$ for all t as a result of (A.14). It follows that

$$P_t^c = Z_t N_t - W_t N_t + l_t^n (1-s) N_t. \tag{A.16}$$

Consider the ex-dividend value of equity $P_t = P_t^c - D_t$. Equation A.16 and the definition of dividends imply

$$\begin{aligned} P_t &= Z_tN_t - W_tN_t + l_t^n(1-s)N_t - Z_tN_t + W_tN_t + \kappa_tV_t \\ &= \kappa_tV_t + l_t^n(1-s)N_t \\ &= \kappa_tV_t + l_t^n(1-s)N_t - l_t^vZ_tq(\theta_t)V_t \\ &= l_t^n((1-s)N_t + q(\theta_t)V_t) \\ &= l_t^nN_{t+1}. \end{aligned} \tag{A.17}$$

where the last three equations follow from (A.10), (A.12), and (A.13). Combining (A.17) with (A.13) results in (A.5).

We now show (A.6). From (A.5) and the definition of dividends, it follows that

$$\begin{aligned} R_{t+1} &\equiv \frac{P_{t+1}+D_{t+1}}{P_t} \\ &= \frac{l_{t+1}^nN_{t+2} + Z_{t+1}N_{t+1} - W_{t+1}N_{t+1} - \kappa_{t+1}V_{t+1}}{l_t^nN_{t+1}} \\ &= \frac{l_{t+1}^n\frac{N_{t+2}}{N_{t+1}} + Z_{t+1} - W_{t+1} - \frac{\kappa_{t+1}V_{t+1}}{N_{t+1}}}{l_t^n} \\ &= \frac{l_{t+1}^n\left[1-s+q(\theta_{t+1})\frac{V_{t+1}}{N_{t+1}}\right] + Z_{t+1} - W_{t+1} - \kappa_{t+1}\frac{V_{t+1}}{N_{t+1}}}{l_t^n} \\ &= \frac{Z_{t+1} - W_{t+1} + l_{t+1}^n(1-s) - l_{t+1}^vZ_{t+1}q(\theta_{t+1})\frac{V_{t+1}}{N_{t+1}}}{l_t^n} \\ &= \frac{Z_{t+1} - W_{t+1} + (1-s)\left[\frac{\kappa_{t+1}}{q(\theta_{t+1})} - l_{t+1}^vZ_{t+1}\right]}{\frac{\kappa_t}{q(\theta_t)} - l_t^vZ_t} \end{aligned} \tag{A.18}$$

where the last equation follows from $l_t^vZ_tq(\theta_t)V_t = 0$.

Using this result, we provide characterizations of returns and prices that will be useful in what follows.

Lemma A.1. *Under the assumptions $\kappa_t = Z_t\kappa$ and $b_t = Z_tb$, the equity return equals*

$$R_{t+1} = \frac{\left[(1-s)\frac{\kappa}{q(\theta_{t+1})} - l_{t+1}^v\right] + 1 - w(\theta_{t+1})}{\frac{\kappa}{q(\theta_t)} - l_t^v}\frac{Z_{t+1}}{Z_t}, \tag{A.19}$$

where $w(\theta_t)$ is the wage normalized by productivity:

$$w(\theta_t) = (1-B)b + B(1+\kappa(\nu\theta_t + (1-\nu)\bar{\theta})). \tag{A.20}$$

The result follows directly from Theorem A.2.1, Equation A.6.

Given l^n_t as the value of a worker inside the firm, the Euler equation (A.14) suggests a notion of a payout of a worker inside the firm:

$$D^l_t = Z_t - W_t - s l^n_t. \tag{A.21}$$

Lemma A.2. *Under the assumptions $\kappa_t = Z_t\kappa$ and $b_t = Z_t b$, the payout ratio of a worker employed in a firm is given by*

$$\begin{aligned} \frac{D^l_t}{l^n_t} &= \frac{Z_t - W_t - s l^n_t}{l^n_t} && \text{(A.22)} \\ &= \frac{1 - w(\theta_t) - s\left[\frac{\kappa}{q(\theta_t)} - l^v_t\right]}{\frac{\kappa}{q(\theta_t)} - l^v_t}. && \text{(A.23)} \end{aligned}$$

Proof Equation A.23 follows directly from (A.13) and (A.21).

How does this notion of payout ratio relate to the more traditional dividend-price ratio?

Lemma A.3. *Consider the dividend-price ratio for the firm, D_t/P_t. Then,*

$$1 + \frac{D_t}{P_t} = \left(1 + \frac{D^l_t}{l_t}\right)\frac{N_t}{N_{t+1}} \tag{A.24}$$

Thus, if the labor market is in a steady state (defined as $N_t = N_{t+1}$), $D_t/P_t = D^l_t/l_t$.

Proof It follows from (A.5), the definition of dividends (A.8), the law of motion for N_t (A.3), and $l^v_t Z_t q(\theta_t)V_t = 0$ that

$$\begin{aligned} P_t + D_t &= l^n_t N_{t+1} + Z_t N_t - W_t N_t - \kappa_t V_t \\ &= \left(Z_t - W_t + l^n_t\left(1 - s + \frac{q(\theta_t)V_t}{N_t}\right) - \frac{\kappa_t V_t}{N_t}\right) N_t \\ &= (Z_t - W_t + l^n_t(1-s))N_t \end{aligned}$$

where the last equation follows from (A.12) and (A.13). Thus

$$
\begin{aligned}
1+\frac{D_t}{P_t} &= \frac{P_t+D_t}{P_t} \\
&= \frac{Z_t-W_t+l_t^n(1-s)}{l_t^n}\frac{N_t}{N_{t+1}} \\
&= \frac{1-w(\theta_t)+(1-s)\left(\frac{\kappa}{q(\theta_t)}-l_t^v\right)}{\frac{\kappa}{q(\theta_t)}-l_t^v}\frac{N_t}{N_{t+1}} \\
&= \left(1+\frac{D_t^l}{l_t^n}\right)\frac{N_t}{N_{t+1}}
\end{aligned}
$$

where the last line follows from (A.22).

Figure 1 shows that the cyclical variation in the vacancy-unemployment ratio implied by (A.5) given the unemployment rate and the price-productivity ratio in the data lines up closely with its counterpart in the data.

B Equilibrium Solution

Let x' denote the value of the variable x in period $t+1$ and x the value at t. The value function and policy functions are functions of the exogenous state variables λ and $\hat{\kappa}$, as well as the endogenous state variable N. The dynamics of the stochastic discount factor and returns are driven by five shocks: disaster probability λ', normal times productivity shock ϵ', disaster indicator d', disaster size ζ', and $\hat{\kappa}'$. Let $\mathbb{E}$ be the expectation operator over four shocks. In our numerical procedure, we solve for the vacancy policy $V(\lambda, N, \hat{\kappa})$ and the Lagrange multiplier on the non-negativity constraint $l^v(\lambda, N, \hat{\kappa})$. The market clearing condition allows us to compute the consumption policy given the vacancy rate: $c(\lambda, N, \hat{\kappa}) = N + b(1-N) - \hat{\kappa}V(\lambda, N, \hat{\kappa})$.

The stochastic discount factor can be written as

$$
\begin{aligned}
M(\lambda, N, \hat{\kappa}; \lambda', \epsilon', d', \zeta', \hat{\kappa}') &= \beta e^{-\frac{\mu}{\psi}+\frac{1}{2}(1-\gamma)\left(\gamma-\frac{1}{\psi}\right)\sigma_\epsilon^2} e^{-\gamma(\epsilon'+d'\zeta')} \\
&\quad \cdot \mathbb{E}\left[e^{(1-\gamma)d'\zeta'} j(\lambda', N', \hat{\kappa}')^{1-\gamma}\right]^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}} \left(\frac{c(\lambda', N', \hat{\kappa}')}{c(\lambda, N, \hat{\kappa})}\right)^{-\frac{1}{\psi}} j(\lambda', N', \hat{\kappa}')^{\frac{1}{\psi}-\gamma},
\end{aligned} \tag{B.1}
$$

where j is the normalized utility function as in Kilic and Wachter (2018). The equity return from

Theorem 1 is given by

$$R(\lambda, N, \hat{\kappa}; \lambda', \epsilon', d', \zeta', \hat{\kappa}') = e^{\mu+\epsilon'+d'\zeta'} \left[\frac{1 - w(\lambda', N', \hat{\kappa}') + (1-s)\left[\frac{\hat{\kappa}'}{q(\theta(\lambda', N', \hat{\kappa}'))} - l^{v'}(\lambda', N', \hat{\kappa}')\right]}{\frac{\hat{\kappa}}{q(\theta(\lambda, N, \hat{\kappa}))} - l^{v}(\lambda, N, \hat{\kappa})} \right], \quad \text{(B.2)}$$

where

$$w(\lambda, N, \hat{\kappa}) = (1-B)b + B(1 + \kappa((1-\nu)\bar{\theta} + \nu\theta(\lambda, N, \hat{\kappa}))) \quad \text{(B.3)}$$

and

$$\theta(\lambda, N, \hat{\kappa}) = \frac{N + b(1-N) - c(\lambda, N, \hat{\kappa})}{\hat{\kappa}(1-N)}, \quad \text{(B.4)}$$

which follows from the market clearing condition.

Equations (A.13) and (A.14) imply that the equilibrium condition that $V(\lambda, N, \hat{\kappa})$, $l^v(\lambda, N, \hat{\kappa})$ and $j(\lambda, N, \hat{\kappa})$ have to satisfy is

$$\mathbb{E}\left[M(\lambda, N, \hat{\kappa}; \lambda', \epsilon', d', \zeta', \hat{\kappa}')R(\lambda, N, \hat{\kappa}; \lambda', \epsilon', d', \zeta', \hat{\kappa}')\right] = 1 \quad \text{(B.5)}$$

and the recursive utility function is given by

$$j(N, \lambda, \hat{\kappa}) = \left[c(N, \lambda, \hat{\kappa})^{1-\frac{1}{\psi}} + \beta e^{\left(1-\frac{1}{\psi}\right)\mu + \frac{1}{2}\left(1-\frac{1}{\psi}\right)(1-\gamma)\sigma_\epsilon^2} \left(\mathbb{E}\left[e^{(1-\gamma)d'\zeta'} j(\lambda', N', \hat{\kappa}')^{1-\gamma} \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}}. \quad \text{(B.6)}$$

We approximate the AR(1) process for log disaster probability by a 14-state Markov process and use the corresponding probability transition matrix to calculate expectations over λ'. The expectations over ζ' and ϵ' can be taken directly since their distributions are *iid*.

We solve for the equilibrium vacancy policy by iterating on the Euler equation (B.5). First, we specify a grid of state variables. The grid for N has 200 equally-spaced nodes from 0.01 to 0.99. We recursively span possible values of $\hat{\kappa}$ starting from $\overline{\kappa}$ and iterating 75 times $(1-\rho_\kappa)\underline{\kappa} + \rho_\kappa\hat{\kappa}$. We further place five nodes between the lowest value from the iteration and $\underline{\kappa}$.

Given the grid of state variables, the computation steps are as follows:

1. Start with an initial guess for the functions V, l^v, and j.
2. Compute next period's endogenous state (N') using the guess as well as probabilities and possible values for the exogenous states ($\hat{\kappa}'$, λ') using the corresponding stochastic processes.
3. Compute next period's vacancy policy (V'), utility function (j') and the Lagrange multiplier ($l^{v'}$)

by linear interpolation on the grid.

4. Given next period's states and policy, calculate $q(\theta)$ assuming $l^v = 0$. If $q(\theta) < 1$, then the constraint $V \geq 0$ is not binding and we set $l^v = 0$. We then solve $\theta = q^{-1}(q(\theta))$, in which $q^{-1}(q(\theta)) = ((q(\theta)^{-\iota}) - 1)^{1/\iota}$ is the inverse of $q(\theta) = (1 + \theta^{\iota})^{-1/\iota}$ and $V = \theta(1 - N)$. If $q(\theta) \geq 1$, then $V \geq 0$ is binding and we set $V = 0$, $\theta = 0, q(\theta) = 1$, and compute the value for l^v that ensures that the Euler equation holds.

5. Compute today's utility j in (B.6) given the consumption policy implied by condition (B.5) and the last iteration's utility values for j'.

6. Repeat 2 - 6 until current consumption policy does not change significantly.

C Model with Capital Accumulation

We solve the extension of the model with capital incorporating the non-negativity constraint for V_t. Let $\tilde{K}_{t+1}$ denote planned capital given by

$$\tilde{K}_{t+1} = K_{t+1} e^{-d_{t+1}\zeta_{t+1}}. \tag{C.1}$$

Note that $\tilde{K}_{t+1}$ is known at time t. Law of motion for capital and (C.1) imply the following law of motion for $\tilde{K}_{t+1}$:

$$\tilde{K}_{t+1} = (1-\delta)\tilde{K}_t e^{d_t\zeta_t} + \phi\left(\frac{I_t}{K_t}\right)\tilde{K}_t e^{d_t\zeta_t}. \tag{C.2}$$

The representative firm pays

$$D_t = F(K_t, Z_t N_t) - W_t N_t - \kappa_t V_t - I_t \tag{C.3}$$

as dividend.

The firm problem with capital is then given by

$$P_t^c = \max_{\{V_{t+\tau}, N_{t+\tau+1}, I_{t+\tau}, \tilde{K}_{t+\tau+1}\}_{\tau=0}^{\infty}} \mathbb{E}_t \sum_{\tau=0}^{\infty} M_{t+\tau}\left[F(K_{t+\tau}, Z_{t+\tau}N_{t+\tau}) - W_{t+\tau}N_{t+\tau} - \kappa_{t+\tau}V_{t+\tau} - I_{t+\tau}\right], \tag{C.4}$$

subject to the law of motion for employment

$$N_{t+1} = (1-s)N_t + q(\theta_t)V_t, \tag{C.5}$$

the non-negativity constraint of V_t

$$F_{N,t}q(\theta_t)V_t \geq 0, \tag{C.6}$$

and the law of motion for planned capital

$$\tilde{K}_{t+1} = (1-\delta)\tilde{K}_t e^{d_t \zeta_t} + \phi\left(\frac{I_t}{K_t}\right)\tilde{K}_t e^{d_t \zeta_t}. \tag{C.7}$$

The first-order conditions with respect to V_t and N_{t+1} are given by

$$\kappa_t = l_t^v F_{N,t} q(\theta_t) + l_t^n q(\theta_t) \tag{C.8}$$

$$l_t^n = \mathbb{E}_t\left[M_{t+1}(F_{N,t+1} - W_{t+1} + l_{t+1}^n(1-s))\right], \tag{C.9}$$

where $F_{N,t+1}$ is the partial derivative of F_{t+1} with respect to N_{t+1}.

The first-order conditions with respect to I_t and $\tilde{K}_{t+1}$ are given by

$$-1 + l_t^k \phi'\left(\frac{I_t}{K_t}\right) = 0 \tag{C.10}$$

$$l_t^k = \mathbb{E}_t\left[M_{t+1}e^{d_{t+1}\zeta_{t+1}}\left(F_{K,t+1} + l_{t+1}^k\left(1-\delta+\phi\left(\frac{I_{t+1}}{K_{t+1}}\right) - \phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}\right)\right)\right], \tag{C.11}$$

where $F_{K,t+1}$ is the partial derivative of F_{t+1} with respect to K_{t+1} and l_t^v is the Lagrange multiplier of the non-negativity constraint.

Recursive substitution of first-order conditions into the firm value in (C.4) imply the equivalent of Theorem 1 for the model with capital:

$$P_t = l_t^k \tilde{K}_{t+1} + l_t^n N_{t+1}, \tag{C.12}$$

where $l_t^k = 1/\phi'\left(\frac{I_t}{K_t}\right)$ and $l_t^n = \kappa_t/q(\theta_t) - l_t^v F_{N,t}$.

Finally, market clearing implies that aggregate consumption is given by

$$C_t = F(K_t, Z_t N_t) + b_t(1-N_t) - \kappa_t V_t - I_t, \tag{C.13}$$

which can be used to compute model dynamics in general equilibrium where $b_t = bF_{N,t}$ and $\kappa_t = \hat{\kappa}_t F_{N,t}$.

We solve the model with capital applying the Euler equation iteration approach described in Section

B. We add $k = \frac{K}{Z}$ (normalized capital) to the grid of state variables (in addition to λ, $\hat{\kappa}$, N), and solve for vacancy and investment policies using the equilibrium conditions (C.13), (C.9) and (C.11).